DYNAMIC IDENTIFICATION OF BUILDING STRUCTURES: NEW STRATEGIES

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Abstract. *In this paper the evolution of a time domain dynamic identification technique based on a statistical moment approach is presented. This technique can be used in the case of structures under base random excitations in the linear state and in the non linear one. By applying Itô stochastic calculus, special algebraic equations can be obtained depending on the statistical moments of the response of the system to be identified. Such equations can be used for the dynamic identification of the mechanical parameters and of the input. The above equations, differently from many techniques in the literature, show the possibility of obtaining the identification of the dissipation characteristics independently from the input. Through the paper the first formulation of this technique, applicable to non linear systems, based on the use of a restricted class of the potential models, is presented. Further a second formulation of the technique in object, applicable to each kind of linear systems and based on the use of a class of linear models, characterized by a mass proportional damping matrix, is described.*

Sommario. *In questo articolo si presenta l'evoluzione di una tecnica di identificazione dinamica basata su un approccio che utilizza i momenti statistici della risposta strutturale. Questa tecnica può essere usata per sistemi lineari e non-lineari eccitati alla base da una forzante random. Attraverso il calcolo differenziale stocastico di Itô è possibile ottenere speciali equazioni algebriche che dipendono dai momenti statistici della risposta del sistema da identificare. Le equazioni algebriche ottenute possono così essere utilizzate per identificare sia i parametri meccanici del sistema sia quelli legati alla forzante. Differentemente da altri metodi presenti in letteratura, la tecnica proposta permette di ottenere i parametri legati alla dissipazione viscosa, indipendentemente dall'input. Nel lavoro vengono presentati la prima formulazione del metodo, basata su una sottoclasse di modelli a potenziale e applicabile anche a sistemi non-lineari e una seconda formulazione che, superando alcuni dei limiti della prima, è basata su una classe di modelli lineari con matrice di dissipazione proporzionale alla massa.*

1. INTRODUCTION

In the last three decades different identification techniques have been formulated based on the dynamic response of the systems to be identified. The first techniques requested the knowledge of the input (in a deterministic or in a probabilistic sense) both in the field of linear system identification[1,2] and in the field of non-linear system identification[3,4,5]. But input is not always simply obtainable as in the case of environmental excitations. On the other hand, the fact of not being necessary to measure the input is an advantage anyway.

In the last years, the interest in developing techniques valid in the case of unmeasurable or unmeasured input, to which this paper is addressed, has increased. In this field, referring to time invariant systems – namely, whose mechanic characteristics can be defined independently from the time and the state variables - some interesting parametric approaches have been proposed in the literature[6,7]. In the field of the non-parametric approaches, the works[8,9] have to be remembered.

In the works referenced above the damping estimation dependence from the characteristics of the input is stressed, evidencing also the connected estimation difficulties. These difficulties increase in the case of hysteretic systems or in general in the case of systems that, because of deterioration, change their mechanical characteristics. In fact, for these systems the knowledge of the input is also requested[4,5].

The actual framework of the research in dynamic identification shows that the improvement of the available techniques or the formulation of new techniques, not depending on input data, is a target to be reached. In the present paper a time domain approach and its evolution is discussed. First the identification of *MDOF* non-linear systems under a unmeasured unknown white noise input is faced, further an implementation of the method explained, devoted to linear systems but with some specific properties that make it better from the computational point of view, is described.

In the first and in the second case the identification procedure consists of three steps. The stiffness and dissipation parameters are obtained respectively in the first and in the second step while the input parameters are obtained in the third one. In each stage *Itô* calculus[10] is used and some analytical manipulations are carried out in order to obtain the solving equations.

For identification purpose, in the first case, a particular class of potential models, referred with the acronyms *RPM*, is used. For this class the energy dissipation depends on the velocities and on a polynomial of the total energy of the system, while the restoring forces may be any type of non-linear function of the displacements to which a potential energy can be associated[11,12]. *RPMs* have been used here mainly for the following reasons: i) their analytical properties make simple the posing of an identification problem; ii) *RPMs* allow one to describe the behaviour of a very wide class of non-linear systems, as it was shown in the works[11,12]; iii) the response of *RPM* in a statistical sense is exactly known, and therefore, once the equations describing the structural behaviour are found, the problem of finding their solution is also solved.

In the second case a class of linear models is used, characterized by a mass proportional damping. Also in this case the structure of the model allows to formulate, in a simple way, the identification algorithm, further, as in the first case, the response in a statistical sense is exactly known, that being an advantage for each following predictive analysis.

2. FIRST FORMULATION: MODELS USED AND ALGORITHM OBTAINED

This formulation is based on the use of *RPMs* whose analytical form is

$$\ddot{\boldsymbol{X}} + \boldsymbol{K}\frac{\partial}{\partial \dot{\boldsymbol{X}}} g\left(H\right) + \boldsymbol{r}(\boldsymbol{X}) = \boldsymbol{W}\,;\; H = h(\boldsymbol{X}, \dot{\boldsymbol{X}}) \tag{1}$$

where $\boldsymbol{X}$ is the N-dimensional displacement vector, the upper dot meaning time derivative, $\boldsymbol{r}(\boldsymbol{X})$ is any non-linear function vector representing the restoring forces, and $\boldsymbol{W}(t)$ (the external input) is a vector of zero mean white noise processes characterized by the correlation matrix $\boldsymbol{R}$ whose ij-th term R_{ij} is

$$R_{ij} = E\left[W_i(t)W_j(t+\tau)\right] = 2\pi K_{ij}\delta(\tau) \tag{2}$$

In Eq.(2) $E[\cdot]$ is the average operator, t means time, τ is a time delay, $\delta(\tau)$ is the Dirac's *delta* and K_{ij} is the ij-th term of the matrix $\boldsymbol{K}$, that is the *Power Spectral Density* (*PSD*) matrix of $\boldsymbol{W}$. In Eq.(1) $h(\boldsymbol{X},\dot{\boldsymbol{X}})$ is the total energy of the system, that is

$$h(\boldsymbol{X},\dot{\boldsymbol{X}}) = \frac{1}{2}\dot{\boldsymbol{X}}^T\dot{\boldsymbol{X}} + U(\boldsymbol{X}) \tag{3}$$

$U(\boldsymbol{X})$ being the potential energy whose partial derivatives are the restoring forces ($r_i(\boldsymbol{X}) = \partial U(\boldsymbol{X})/\partial X_i$), $\frac{\partial}{\partial \dot{\boldsymbol{X}}}$ is a velocity gradient operator, that is $\frac{\partial}{\partial \dot{\boldsymbol{X}}}^T = \left[\frac{\partial}{\partial \dot{X}_1},......,\frac{\partial}{\partial \dot{X}_2}\right]$, finally $g(\cdot)$ is a non-linear function.

The second term in the left side of Eq.(1) constitutes a vector of dissipation forces depending on some invariant parameters and the *PSD* matrix $\boldsymbol{K}$. For the aim of this study a polynomial form of the function $g(H)$ has been fixed, that is

$$g(H) = \pi\sum_{j=1}^{s} a_j\left[h(\boldsymbol{x},\dot{\boldsymbol{x}})\right]^j \tag{4}$$

where a_j (j=1,…,s) are invariant parameters.

By some analytical manipulations[13] and by applying the *Itô* calculus the following equations can be obtained, respectively, for the identification of the stiffness parameters and of the dissipation parameters

$$E\left[\ddot{\boldsymbol{X}}X_i^{2k-1}\right] + E\left[\boldsymbol{r}(\boldsymbol{X})X_i^{2k-1}\right] = \boldsymbol{0} \tag{5}$$

$$-\sum_{j=1}^{s} ja_j E\left[H^{m+j-2}\dot{X}_i^2\right] + E\left[H^{m-1}\right] + (m-1)E\left[H^{m-2}\dot{X}_i^2\right] = 0 \tag{6}$$

These equations describe a set having as coefficients the averages of some functions of the response and as unknowns the parameters defining the restoring forces and the dissipation forces. Hence Eq.(5) and Eq.(6) can be used once the above averages are evaluated by processing the system response. Eq.(5) and Eq.(6) do not depend on the parameters that define the input, this fact constituting a simplification in the identification problem. Nevertheless Eq.(6) is not sufficient for the identification of the damping forces depending also, as evidenced in Eq.(1), on the *PSD* matrix of the input. Hence a complete estimation of the dissipation forces depends on the identification of the matrix $\boldsymbol{K}$. Further analytical manipulations[13] allow to obtain the following equation for the identification of each term of the matrix $\boldsymbol{K}$

$$E\left[\dot{X}_q\ddot{X}_i^+\right] = -\pi K_{iq} \tag{7}$$

where the signum (+) means that the quantity $\ddot{X}_i^+$ is shifted of dt with respect of $\dot{X}_q$. The details regarding the strategies for obtaining Eq.(5) and Eq.(6) can be found in the papers[13,14].

As already mentioned, once the model parameters have been identified the response of the system can be obtained, in a statistical sense, on the base of the knowledge of the exact expression of the probability density function (*pdf*), that is

$$p_{X,\dot{X}}(\boldsymbol{x},\dot{\boldsymbol{x}}) = c\exp\left(-\frac{1}{\pi}g\left(h(\boldsymbol{x},\dot{\boldsymbol{x}})\right)\right);$$

$$\frac{1}{c} = \int_{-\infty}^{+\infty} ..... \int_{-\infty}^{+\infty} \exp\left(-\frac{1}{\pi}g\left(h(\boldsymbol{x},\dot{\boldsymbol{x}})\right)\right) dx_1 \ldots dx_n d\dot{x}_1 \ldots d\dot{x}_n \tag{8}$$

Observe that this *pdf* gives an equal distribution in probability of the velocities in each degree of freedom. This fact does not prevent the use for systems that are featured by a different distribution in probability of the velocities as it will be clarified in the next section.

3. FIRST FORMULATION: COMPUTER SIMULATION

The response was generated by means of the following three-degrees-of-freedom model

$$\ddot{\boldsymbol{X}} + \alpha\, \boldsymbol{f}\left(\dot{\boldsymbol{X}}\right) + \boldsymbol{r}\left(\boldsymbol{X}\right) = \boldsymbol{W};\ \left[\boldsymbol{f}\left(\dot{\boldsymbol{X}}\right)\right]^T = \left[\dot{X}_1^3, \dot{X}_2^3, \dot{X}_3^3\right];$$
$$\left[r(X)\right]^T = \left[b_1 X_1^3 + b_2 X_2 + b_3 X_3;\ c_1 X_1 + c_2 X_2^3 + c_3 X_3;\ d_1 X_1 + d_2 X_2 + d_3 X_3^3\right] \tag{9}$$

where $\boldsymbol{r}(\boldsymbol{X})$ is a restoring force vector, α is a dissipation time invariant parameter that was assumed equal to 0.005. The values of the parameters of the restoring forces and the entries of the *PSD* matrix of the input (k_{ij}) were fixed as follows

$$b_1 = 200;\ b_2 = -100;\ b_3 = 0;\ c_1 = -100;\ c_2 = 200;\ c_3 = -100;$$
$$d_1 = 0;\ d_2 = -100;\ d_3 = 100;\ K_{ij} = K_0 = 100\ \forall i, j \tag{10}$$

It is simply recognizable that Eq.(9) does not belong to the class of *RPMs* and does not give the same statistics for the velocities in each degree-of-freedom. Once the time history of the input was generated by the *PSD* matrix, that refers to a base excitation, the system response was calculated by the fourth order Runge Kutta integration of Eq.(9) and was processed by the identification algorithm.

For the identification, the following *RPM* was selected

$$\ddot{\boldsymbol{X}} + \pi\left(\hat{a}_1 + 2\hat{a}_2 H\right)\hat{\boldsymbol{K}}\dot{\boldsymbol{X}} + \hat{\boldsymbol{r}}\left(\boldsymbol{X}\right) = \boldsymbol{W} \tag{11}$$

where $\hat{\boldsymbol{r}}\left(\boldsymbol{X}\right)$ and the entries $\hat{K}_{ij}$ of $\hat{\boldsymbol{K}}$ were assumed to have the following form

$$\overline{\boldsymbol{r}}\left(\boldsymbol{X}\right) = \hat{b}_1 X_1^3 + \hat{b}_2 X_2 + \hat{b}_3 X_3;\ \hat{c}_1 X_1 + \hat{c}_2 X_2^3 + \hat{c}_3 X_3;\ \hat{d}_1 X_1 + \hat{d}_2 X_2 + \hat{d}_3 X_3^3;\ \hat{k}_{ij} = \hat{K}_0 \tag{12}$$

Now $\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2, \hat{b}_3, \hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{d}_1, \hat{d}_2, \hat{d}_3, \hat{K}_0$ are coefficients to be estimated. The algorithm will be effective if, at the end of the identification procedure, the following identities are obtained

$$\hat{b}_1 = b_1,\ \hat{b}_2 = b_2,\ \hat{b}_3 = b_3,\ \hat{c}_1 = c_1,\ \hat{c}_2 = c_2,\ \hat{c}_3 = c_3,\ \hat{d}_1 = d_1,\ \hat{d}_2 = d_2,\ \hat{d}_3 = d_3;\ \hat{K}_0 = K_0 \tag{13}$$

Referring to the *PSD* matrix of the input, because of the different distribution in probability of the velocities of the “real system”, the estimation of $\hat{K}_{ij}$ by Eq.(7) were further processed: it was proved that a good estimation of $\hat{K}_0$ could be obtained using the results of Eq.(7) itself by means of the following equation

$$\frac{\sum_{i=1,j=1}^{N,N} \hat{k}_{ij}}{N \cdot N} = \hat{K}_0 \tag{14}$$

Once the parameters $\hat{a}_1, \hat{a}_2$ were estimated, the energy moment of the "real system", depending on the variance of the velocities in each degree-of-freedom, could be compared to the energy moment obtainable by the *pdf* of the *RPM* used for the identification, so to verify the suitability of the dissipation parameters obtained. In Figure 1 the results of the estimation of the stiffness parameter are inserted while in Figure 2 the parameter defining the input, K_0, and the comparison between the average of the energy of the identified system, obtained from its response, and the average of the energy of the identifying model, calculated by its *pdf*, are inserted.

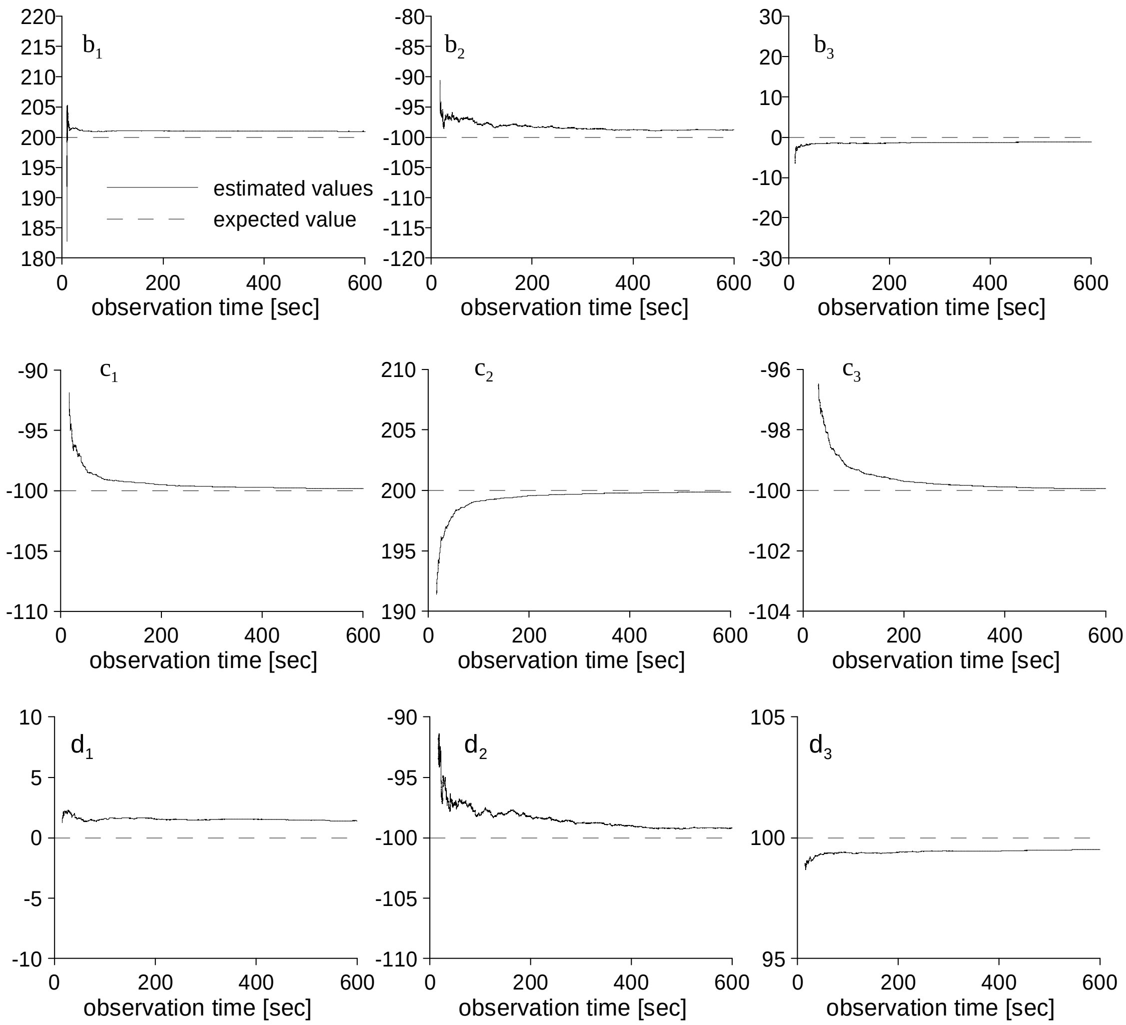

Figure 1: First formulation: estimation of the stiffness parameters

The estimations are made at each instant during an observation time of 600 sec. Figures 1 and 2 show that a good estimation is possible after few seconds of observation, that is basic for a system that is supposed to be time invariable.

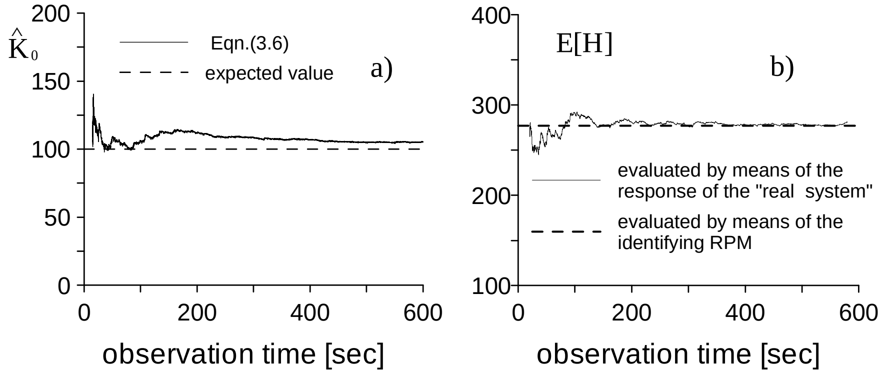

Figure 2: First formulation: estimation of the input parameter (a) and evaluation of the average of the energy by the identifying model (b)

4. SECOND FORMULATION: MODELS USED AND ALGORITHM OBTAINED

Now the attention is focused on a restricted class of *MDOF* linear models that can be described by the following relationship

$$\boldsymbol{M}\ddot{\boldsymbol{X}} + \boldsymbol{D}\dot{\boldsymbol{X}} + \boldsymbol{S}\boldsymbol{X} = \boldsymbol{W} \tag{15}$$

where $\boldsymbol{M}$ is the *NxN* diagonal mass matrix (*N* is the number of degrees of freedom modelled by Eq.(15)), $\boldsymbol{D}$ is a damping matrix and $\boldsymbol{S}$ is the stiffness matrix, while $\boldsymbol{W}$ assume the significance specified above. Let the model (15) refer to classically damped systems with distinct un-damped natural frequencies and $\boldsymbol{D}$ simply proportional to the matrix $\boldsymbol{M}$, that is

$$\boldsymbol{D} = \alpha \boldsymbol{M} \tag{16}$$

Taking Eq.(16) into account, in the case of base excitation, Eq.(15) can be rewritten in the form:

$$\boldsymbol{M}\ddot{\boldsymbol{X}} + \alpha \boldsymbol{M}\dot{\boldsymbol{X}} + \boldsymbol{S}\boldsymbol{X} = \boldsymbol{M}\boldsymbol{L}W_0 \tag{17}$$

$\boldsymbol{L}$ being the N-dimensional vector assuming the form $\boldsymbol{L}^T = [1,1,........,1]$ and W_0 the white noise base input whose power spectral density is K_0 . By multiplying both sides of Eq.(17) by $\boldsymbol{M}^{-1}$ one obtains

$$\ddot{\boldsymbol{X}} + a\dot{\boldsymbol{X}} + \boldsymbol{S}^*\boldsymbol{X} = \boldsymbol{L}W_0\,;\;\; \boldsymbol{S}^* = \boldsymbol{M}^{-1}\boldsymbol{S} \tag{18}$$

By some analytical manipulations and by applying the *Itô* calculus the following equations can be obtained, respectively, for the identification of the stiffness parameters, of the dissipation parameter and of the input parameter[15]

$$E\left[\ddot{\boldsymbol{X}}X_i\right] + \boldsymbol{S}^*E\left[\boldsymbol{X}X_i\right] = \boldsymbol{0} \tag{19}$$

$$E\left[\dot{X}_i\ddot{X}_i^+\right] = -\alpha\, E\left[\dot{X}_i^2\right] \tag{20}$$

$$\sum_{i=1}^{N} E\left[\dot{X}_i\ddot{X}_i^+\right] = -\pi\, N\, K_0 \tag{21}$$

5. SECOND FORMULATION: COMPUTER SIMULATION

The response was generated by means of the following three-degrees-of-freedom shear building model

$$\boldsymbol{M}\ddot{\boldsymbol{X}} + \alpha \boldsymbol{M}\dot{\boldsymbol{X}} + \boldsymbol{S}\boldsymbol{X} = \boldsymbol{M}\boldsymbol{L}W_0 \tag{22}$$

where $\boldsymbol{M}$ is the diagonal matrix in which each diagonal term has the value of $2\cdot 10^4$, while $\boldsymbol{S}$ is the stiffness matrix having the following components

$$\boldsymbol{S} = \begin{bmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ d_1 & d_2 & d_3 \end{bmatrix} = \begin{bmatrix} 4\cdot 10^6 & -2\cdot 10^6 & 0 \\ -2\cdot 10^6 & 4\cdot 10^6 & -2\cdot 10^6 \\ 0 & -2\cdot 10^6 & 2\cdot 10^6 \end{bmatrix} \tag{23}$$

Eq.(22) can be rewritten in the form

$$\ddot{\boldsymbol{X}} + \alpha\dot{\boldsymbol{X}} + \boldsymbol{S}^*\boldsymbol{X} = \boldsymbol{L}W_0$$

$$\boldsymbol{S}^* = \boldsymbol{M}^{-1}\boldsymbol{S} = \begin{bmatrix} \hat{b}_1 & \hat{b}_2 & \hat{b}_3 \\ \hat{c}_1 & \hat{c}_2 & \hat{c}_3 \\ \hat{d}_1 & \hat{d}_2 & \hat{d}_3 \end{bmatrix} = \begin{bmatrix} m_1^{-1} & & \\ & m_2^{-1} & \\ & & m_3^{-1} \end{bmatrix} \begin{bmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ d_1 & d_2 & d_3 \end{bmatrix} = \begin{bmatrix} 200 & -100 & 0 \\ -100 & 200 & -100 \\ 0 & -100 & 100 \end{bmatrix} \tag{24}$$

in order to obtain a mass matrix normalized form to which the identification algorithm refers.

Three different simulations were performed, characterized by different values of the dissipation parameter α and of the input parameter K_0 . For the identification, the following linear model was selected

$$\ddot{\boldsymbol{X}} + \tilde{\alpha}\dot{\boldsymbol{X}} + \tilde{\boldsymbol{S}}^*\boldsymbol{X} = \tilde{\boldsymbol{W}}; \quad \tilde{\boldsymbol{S}}^* = \begin{bmatrix} \tilde{b}_1 & \tilde{b}_2 & \tilde{b}_3 \\ \tilde{c}_1 & \tilde{c}_2 & \tilde{c}_3 \\ \tilde{d}_1 & \tilde{d}_2 & \tilde{d}_3 \end{bmatrix} \tag{25}$$

where $\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \tilde{c}_1, \tilde{c}_2, \tilde{c}_3, \tilde{d}_1, \tilde{d}_2, \tilde{d}_3$ were the coefficients to be estimated. The *PSD* of the base excitation, to be estimated, was $\tilde{K}_0$. Clearly, the algorithm will be effective if, at the end of the identification procedure, the following identities are obtained:

$$\begin{gathered} \tilde{b}_1 = \hat{b}_1, \tilde{b}_2 = \hat{b}_2, \tilde{b}_3 = \hat{b}_3, \tilde{c}_1 = \hat{c}_1, \tilde{c}_2 = \hat{c}_2, \tilde{c}_3 = \hat{c}_3, \\ d_1 = \hat{d}_1, \tilde{d}_2 = \hat{d}_2, \tilde{d}_3 = \hat{d}_3, \tilde{K}_0 = K_0, \tilde{\alpha} = \alpha \end{gathered} \tag{26}$$

In the next Figures the results obtained in one of the simulations are shown.

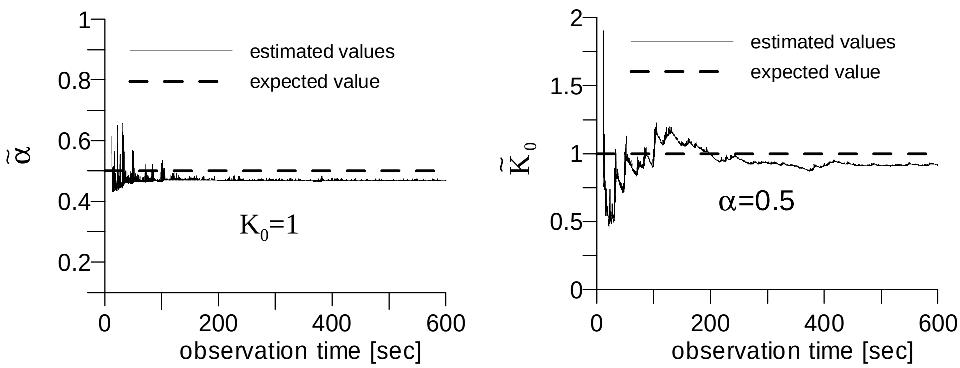

Figure 3: Estimation of the dissipation parameter (a) and of the input parameter (b)

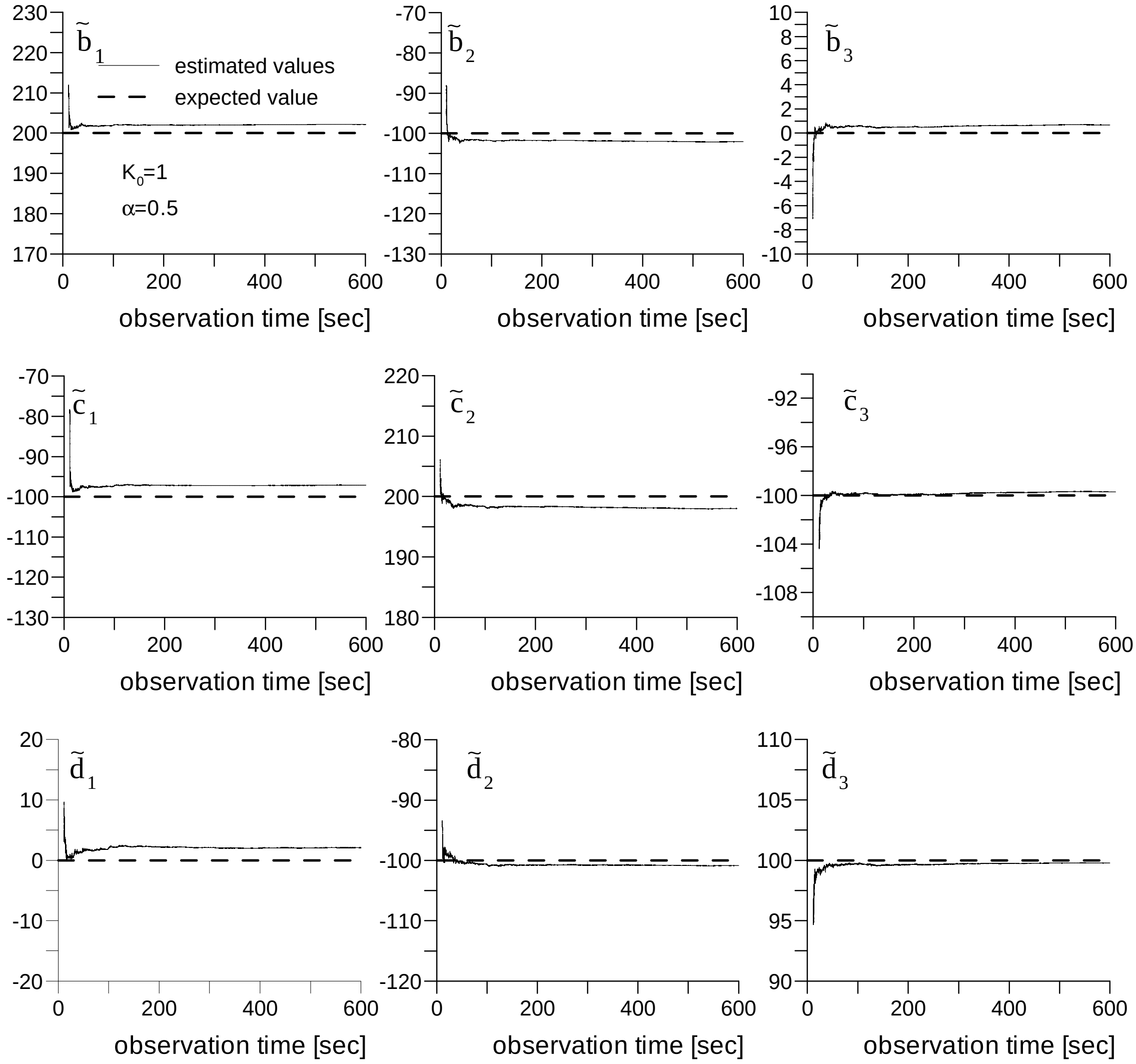

Figure 4: Estimation of the stiffness parameters

6. CONCLUSIONS

Two formulations of an identification technique based on a statistic moment approach have been discussed. The above technique is suitable for civil structures under a base excitation that can be modelled as a white noise.

The first formulation uses a class of potential models characterized by an equal probabilistic distribution of the velocities in each degree of freedom but is usable, thanks to proper manipulations, for the identification of systems that, as it usually happens, are characterized by a different probability distribution of the velocities. An application shows the possibility of obtaining a good estimation for the stiffness parameters, for the input characteristics and for the dissipation ones. Moreover the suitability of this formulation for non linear systems has been proved.

The second formulation of the technique is proper for linear systems and is based on linear models with a mass proportional damping. An application shows the capacity of obtaining a good estimation of the stiffness and dissipation parameters and of the input characteristics.

The technique discussed has the advantage of being applicable in the case of unmeasured or unmeasurable input. Further the algorithms of the first and of the second formulation evidence the possibility of estimating dissipation and input parameters by uncoupled equations differently from the analytical procedures most frequently proposed in the literature.

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