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A NEW PERSPECTIVE ON SEISMIC INTENSITY MEASURES

M.Grigoriu*

*Department of Civil and Environmental Engineering
Cornell University
e-mail: MDG12@cornell.edu

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Abstract. *Concepts of the multivariate extreme value theory (MEVT) are used to estimate the dependence between structural demand parameters D and response spectrum $S_a(T)$, a popular seismic intensity measure (IM). It is assumed that the seismic ground acceleration A(t) is a Gaussian process with known probability law. Under this seismic hazard, the random variables D and $S_a(T)$ are weakly depend so that the usefulness of fragilities defined as functions of $S_a(T)$ is questionable.*

1 INTRODUCTION

Fragilities, i.e., probabilities that structural systems enter various damage states under seismic hazards of specified intensities, are essential tools of performance-based earthquake engineering. Intensity measures, e.g., peak ground acceleration, peak ground velocity, and single/multiple ordinates of the pseudo-acceleration response spectrum $S_a(T)$ for selected periods *T*, are used to characterize the intensity of seismic events. Fragilities are plots of, e.g., damage probabilities versus intensity measures.

Intensity measures have been studied extensively. It was shown that they must be efficient, i.e., structural demand parameters conditional on IMs should have small variances, and sufficient, i.e., the distributions of structural damages should be completely defined for given IMs [*1,7,8,9*]. Efficiency and sufficiency are strong requirements which are rarely if ever satisfied in practice.

We show that structural demand parameters *D* are weakly related to the IM $S_a(T)$ so that fragilities defined as conditional probabilities that *D* exceeds critical values given $S_a(T)$ have large uncertainties. This means that $S_a(T)$ is not an efficient IM and that estimates of seismic performance of structures based on fragilities defined in this manner are likely to be unreliable.

2 INTENSITY MEASURES

We assume that the seismic ground acceleration *A(t)* is a stochastic process with known probability law and limit our discussion to intensity measures defined by single ordinate of the pseudo-acceleration response spectrum $S_a(T)$. Under the assumption that the law of *A(t)* is known, it is possible to calculate statistics of $S_a(T)$ and structural demand parameters *D* exactly and assess the potential of $S_a(T)$ as indicators of site seismic hazard. It is recognized that the setting considered in this study is somewhat constrained and cannot be extended directly to actual ground motions. However, it provides a theoretical framework for assessing IMs which is beneficial to practice.

Various models can be considered for the seismic ground acceleration process *A(t)*. We use the seismological model in [11] to characterize *A(t)*. According to this model, *A(t)* is a stationary Gaussian process with mean zero and spectral density depending on site-to-source distance, moment magnitude, and other physical parameters which is modulated by a deterministic function of time.

Suppose a model has been selected for *A(t)*. Let $X_{sdof}(t)$ denote the displacement of a linear single degree of freedom (SDOF) with natural period T and selected damping ratio. The IM is the largest displacement during the seismic event scaled by the square of the oscillator natural frequency, i.e., the pseudo-acceleration response spectrum $S_a(T) = (2\pi/T)^2 \max_{0\le t\le\tau} \left|X_{sdof}(t)\right|$, where τ denotes the duration of the seismic event. Let *X(t)* denote the response of an arbitrary structural system to *A(t)*. Generally, the system has a large number of degrees of freedom so that *X(t)* is a vector-valued stochastic process. Moreover, *X(t)* is a non-Gaussian process since structural systems do not remain linear and elastic during large seismic events. For simplicity, we consider real-valued demand parameters $D = \max_{0\le t\le\tau} \left|h(X(t))\right|$, where *h* maps *X(t)* into a real-valued quantity of interest, e.g., an interstory displacement or a floor acceleration.

The random variables $S_a(T)$ and *D* are dependent since they are functionals of the same process, the seismic ground acceleration *A(t)*. Current fragilities analysis is based on the hypothesis that the dependence between $S_a(T)$ and *D* is sufficiently strong such that the variance of the conditional random variable $D \mid S_a(T)$ is small. It is shown that this hypothesis does not hold in the framework of this study, a result which question the usefulness of current fragilities as tools for performance-based earthquake engineering. Concepts of the random vibration and multivariate extreme value theories are used to quantify the dependence between $S_a(T)$ and *D*.

3 RELATIONSHIP BETWEEN $S_a(T)$ AND D

Suppose the seismic ground acceleration *A*(*t*), 0≤*t*≤τ, is a stochastic process defined on a probability space (Ω,*F*,*P*). The structural demand *D* and the intensity measure $S_a(T)$ are functionals of the seismic ground acceleration process *A*(*t*), see Eq. (1). The dependence between the random variables $S_a(T)$ and *D* cannot be obtained analytically even in our setting since, although the law of the seismic acceleration process $A(t)$ is known, $S_a(T)$ and D are maxima of filtered versions of $A(t)$, i.e., the stochastic processes $X_{sdof}(t)$ and $X(t)$.

$$\begin{aligned} &A(t) \Rightarrow Structure \Rightarrow X(t) \Rightarrow D = \max_{0\le t\le\tau} \left| h\left(X(t)\right) \right| \\ &A(t) \Rightarrow SDOF \Rightarrow X_{sdof}(t) \Rightarrow S_a(T) = (2\pi/T)^2 \max_{0\le t\le\tau} \left| X_{sdof}(t) \right| \end{aligned} \tag{1}$$

We note that the processes $X_{sdof}(t)$ and $X(t)$ and the random variables $S_a(T)$ and D are defined on the same probability space as $A(t)$ as measurable mappings of it.

3.1 Intuition

Intuition suggests that $S_a(T)$ and D are weakly dependent since they are maxima of two very different processes, a real-valued Gaussian process $X_{sdof}(t)$ whose most energy is concentrated in vicinity of T and a non-Gaussian vector-valued stochastic process $X(t)$ whose frequency content depends strongly on the properties of the particular structural system under consideration and may be very different from the frequency content of $X_{sdof}(t)$. These observations and the fact that processes which have few frequencies in common are weakly correlated suggest that $S_a(T)$ and D are weakly dependent.

That processes with different frequencies are uncorrelated results by simple arguments. For example, let $Y(t)$ be a real-valued weakly stationary process with mean zero and one-sided spectral density $g(\nu)$, $\nu\geq 0$. The processes $Y_1(t)$ and $Y_2(t)$ defined by the harmonics of $Y(t)$ in the frequency bands $[0,\bar{\nu}]$ and $(\bar{\nu},\infty)$ are uncorrelated since they admit the spectral representations $Y_1(t)=\int_0^{\bar{\nu}}\left[\cos(\nu t)dU(\nu)+\sin(\nu t)dV(\nu)\right]$ and $Y_2(t)=\int_{\bar{\nu}}^{\infty}\left[\cos(\nu t)dU(\nu)+\sin(\nu t)dV(\nu)\right]$ which imply $E\left[Y_1(s)Y_2(s)\right]=0$ since $Y(t)=\int_0^{\infty}\left[\cos(\nu t)dU(\nu)+\sin(\nu t)dV(\nu)\right]$ is the spectral representation of $Y(t)$, $E\left[dU(\nu)dU(\nu')\right]=E\left[dV(\nu)dV(\nu')\right]=\delta(\nu-\nu')g(\nu)d\nu$, and $E\left[dU(\nu)dV(\nu')\right]=0, \nu,\nu'\geq 0$.

3.2 Correlation

Translation processes have been proven useful in a broad range of applications in wind/earthquake engineering and material science [2,3,6]. Yet, they are not informative when dealing with simultaneous extremes of dependent random variables, e.g., the random variables $S_a(T)$ and D.

Let $Y=(Y_1,Y_2)$ be a two-dimensional translation random vector, where $Y_k = F_k^{-1}\circ\Phi(G_k)$, k=1,2, $\{F_k\}$ are the marginal distributions of Y, Φ denotes the distribution of the standard Gaussian variable $N(0,1)$, and $\{G_k\}$ are correlated Gaussian variables with zero means and unit variances. Let $\rho = E[G_1 G_2]$ be the correlation of G_1 and G_2. Generally, the correlation between Y_1 and Y_2 is similar to that between G_1 and G_2 [4] (Chap. 3). The joint distribution of the translation variables $Y = (Y_1, Y_2)$ can be given in the form

$$P(Y_1 \le y_1, Y_2 \le y_2) = P\left(F_1^{-1} \circ \Phi(G_1) \le y_1, F_2^{-1} \circ \Phi(G_2) \le y_2\right) = P(G_1 \le u_1, G_2 \le u_2) \quad (2)$$

for any thresholds $\{y_k\}$ and their images l $\{u_k = \Phi^{-1} \circ F_k(z_k)\}$, k=1,2, in the Gaussian space. The correlation coefficient is an attractively simple but crude metric of dependence particularly when the interest is in simultaneous large values of random variables. For example, the joint distribution $P(G_1 \le u_1, G_2 \le u_2)$ of the Gaussian vector (G_1, G_2), |ρ|<1, can be approximated by $\Phi(u_1)\Phi(u_2)$ for sufficiently large levels $\{u_k\}$ since

$$\left|P(G_1 \le u_1, G_2 \le u_2) - \Phi(u_1)\Phi(u_2)\right| \to 0,\ u_1, u_2 \to \infty \quad (3)$$

by the normal comparison lemma [10] (Theorem 4.2.1). A similar statement holds for the non-Gaussian image of (G_1, G_2), i.e., $P(Y_1 \le y_1, Y_2 \le y_2) \simeq F_1(y_1) F_2(y_2)$ for large $\{y_k\}$ and correlation coefficients |ρ|<1.

These considerations show that (1) the translation model is inadequate for the bivariate random vector $(S_a(T), D)$ if the interest is in simultaneous large values of $S_a(T)$ and D and (2) metrics finer than correlation need to be used to determine whether or not simultaneous large values of $S_a(T)$ and D are dependent. Such metrics are considered next.

3.3 Multivariate extreme value theory

Our objective is to estimate the dependence between simultaneous large values of $Y_1 = S_a(T)$ and $Y_2 = D$, i.e., determine whether structural demand parameters can be predicted with satisfactory accuracy from IMs such as $S_a(T)$ particularly for large seismic events. Such events are likely to cause structural damage and/or failure. We use tools of the multivariate extreme value theory (MEVT) to achieve this objective.

Let $\{y_i = (y_{i,1}, y_{i,2})\}, i = 1, \ldots, n$, be n independent samples of $Y = (Y_1, Y_2)$. They are calculated from samples $X_{sdof}(t, \omega)$ and $X(t, \omega)$ of the responses of linear SDOF and structural systems subjected to independent samples $A(x, \omega)$ of the seismic ground acceleration process $A(t)$. Following is a heuristic presentation of MEVT concepts which are relevant to our discussion.

Independent identically distributed components: Suppose $Y_1, Y_2 > 0$ almost surely (a.s.) and have the same distributions, i.e., $F_1 = F_2$, which means that they have identical scales. In polar coordinates, the bivariate vector $Y = (Y_1, Y_2)$ has the form

$$Y = (Y_1, Y_2) = \left(V \cos(\Theta), V \sin(\Theta)\right) \quad (4)$$

where $V=\|Y\|=\left(Y_1^2+Y_2^2\right)^{1/2}$ and $\Theta=\tan^{-1}\left(Y_2/Y_1\right)$. Similarly, the samples of Y admit the representation

$$y_i=\left(y_{i,1},y_{i,2}\right)=\left(v_i\cos\left(\theta_i\right),v_i\sin\left(\theta_i\right)\right),\ i=1,\ldots,n \tag{5}$$

with the notations in Eq. (4). Samples of Y with distances to the origin larger than $v_0>0$ are of interest since they correspond to components of Y which are simultaneous large. The value used for v_0 is critical to capture properties of the right tails of Y_1 and Y_2 [12]. If v_0 is too large, only few samples will be available to estimate the dependence between the right tails of Y_1 and Y_2 so that the resulting estimates will have large variances. If v_0 is insufficiently large, a significant fraction of samples used in the analysis will be in the body of the distributions of Y_1 and Y_2 so that estimates of tail features will be unsatisfactory.

Let $\left\{y_{i_1},\ldots,y_{i_m}\right\}$ denote the subset of $\left\{y_i,\ldots,y_n\right\}$, $m\leq n$, such that $v_{i_j}>v_0$, $j=1,\ldots,m$, where $v_{i_j}=\left\|y_{i_j}\right\|$. The histogram $h(\theta)$ of $\left\{\theta_{i_1},\ldots,\theta_{i_m}\right\}$ corresponding to samples of Y with norm larger than v_0, referred to as *angular measure*, is used to characterize the dependence between the simultaneous large values of Y_1 and Y_2. The support of this histogram is $[0,\pi/2]$. If most of the mass of $h(\theta)$ is concentrated at $\theta=0$ and $\pi/2$, it is unlikely that Y_1 and Y_2 are simultaneous large. In this case, simultaneous large values of Y_1 and Y_2 are nearly independent. If most of the mass of $h(\theta)$ is concentrated at a particular value of θ, it is highly probable that Y_1 and Y_2 are simultaneous large.

Let $Y_k=\lambda G_0^2+\left(1-\lambda\right)G_k^2$, $k=1,2$, where G_0, G_1, and G_2 are independent standard Gaussian variables. Then, Y_1 and Y_2 are identically distributed, perfectly dependent for $\lambda=1$, and independent for $\lambda=0$. The left panels of Fig. 1 show scatter plots of the samples of $Y=\left(Y_1,Y_2\right)$ with norm larger than v_0 extracted from $n=100000$ independent samples of Y. The top, middle, and bottom panels correspond to $\lambda=0.9$, 0.5, and 0.1 and thresholds $v_0=16$, 10, and 12. The right panels show the corresponding angular measures. They show that the components of Y are strongly and weakly dependent for $\lambda=0.9$ and 0.1.

Arbitrary components: Generally, the components Y_1 and Y_2 of Y have different distributions and scales, e.g., the random variables $S_a\left(T\right)$ and D. If the marginal distributions of Y are known, then its components and samples can be scaled to have the same properties so that the previous formulation applies. In most applications the marginal distributions of Y are not know and there are insufficient samples to estimate them. An alternative approach, referred to as the ranks method can be used to construct estimates of angular measures as in Fig. 1. The marginal distributions of Y is not needed to implement this method. Samples of Y are ranked and used to construct estimators of the angular measure which are used to quantify the dependence between simultaneous large values of Y_1 and Y_2, see [13] (Chap. 9) for the theoretical framework of the ranks method and [5] for its application to seismic hazard.

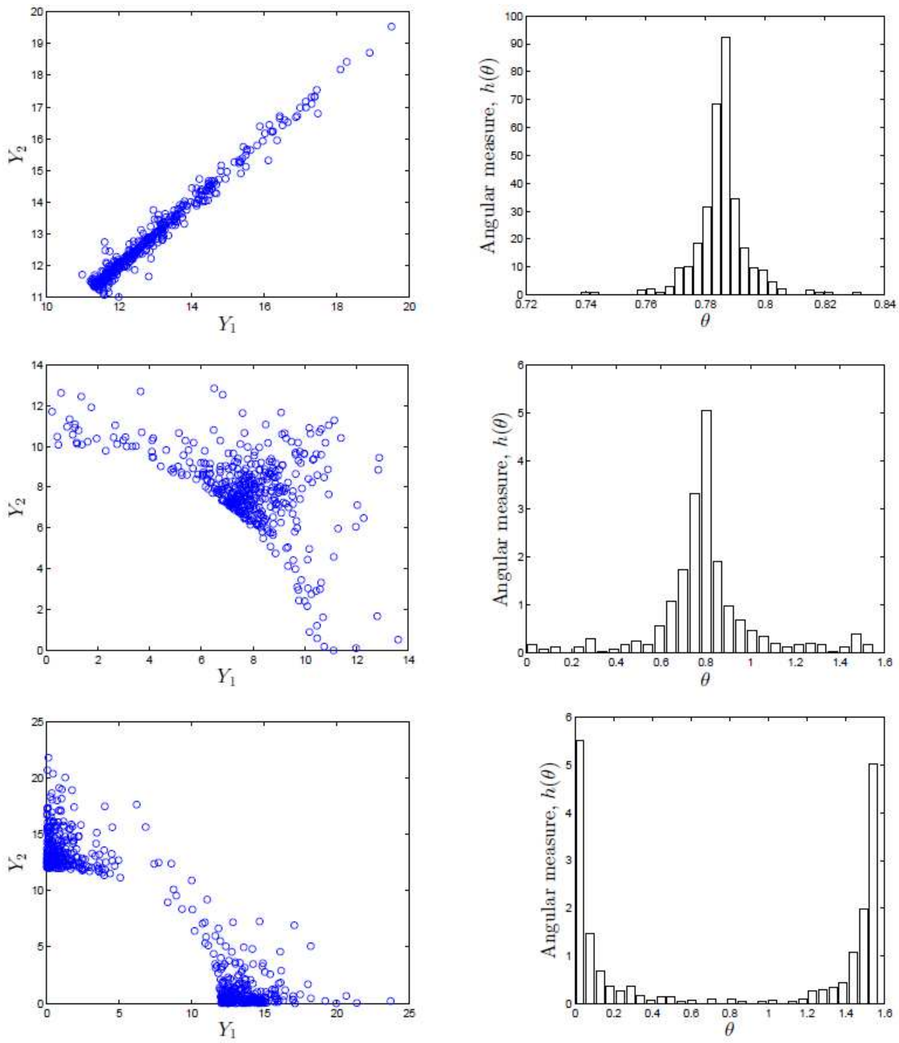

Figure 1. Sample with norms larger than v_0 (left panels) and corresponding angular measures (right panels) for $(\lambda, v_0) = (0.916)$, (0.5,10), and (0.1,12) (top, middle, and bottom panels)

4 DEPENDENCE OF $S_a(T)$ AND D

Suppose the seismic ground acceleration $A(t)$ is a zero-mean Gaussian process with properties given by the seismological model in [11]. Structural responses $X(t)$ to $A(t)$ are non-Gaussian processes because of structures exhibit nonlinear behavior during seismic events. For example, suppose $X(t)$ is the displacement of a nonlinear oscillator and $D=\max_{0\le t\le \tau}|X(t)|$ is the demand parameter. Then, $X(t)$ is a non-Gaussian process and D is the maximum of this process. In contrast, the displacement $X_{sdof}(t)$ of a linear oscillator subjected to $A(t)$ is a Gaussian process and $S_a(T)$ is its maximum. The differences between properties of $X(t)$ and $X_{sdof}(t)$ increase with the degree of nonlinearity/complexity of structural systems and earthquake intensity and so do differences between $S_a(T)$ and D.

Let $X(t)$ be the displacement of a Bouc-Wen oscillator which is defined by

$$\ddot{X}(t)+2\zeta\nu_0\dot{X}(t)+\nu_0^2\left(\rho X(t)+(1-\rho)W(t)\right)=-A(t),\ \text{where}$$
$$\dot{W}(t)=\gamma\dot{X}(t)-\alpha\left|\dot{X}(t)\right|\left|W(t)\right|^{\chi-1}W(t)-\beta\dot{X}(t)\left|W(t)\right|^{\chi} \quad (6)$$

$\nu_0>0$, $\zeta\in(0,1)$, $(\alpha,\beta,\gamma,\rho,\chi)$ are positive constants, and $A(t)$ denotes the seismic acceleration process. The system is linear for $\rho=1$. It nonlinear for $\rho\neq1$ and its behavior depends strongly on the values of β and γ.

The following numerical results are for $\nu_0=2\pi$, $\zeta=0.05$, $\alpha=0.5$, $\beta=5$, $\gamma=3$, $\rho=0.1$, and $\chi=1$. Samples of $X_{sdof}(t)$ and $X(t)$ have been calculated from $n=10000$ independent samples of $A(t)$ and used to calculate the corresponding samples of $S_a(T)$ and D. The resulting samples of the bivariate random vector $(S_a(T),D)$ have been used to estimate angular measures. Calculations used the ranks method since the distributions of $S_a(T)$ and D are unknown.

The left panel of Fig.2 shows the available samples of $(S_a(T),D)$. The circles indicate the samples used to estimate the angular measure $h(\theta)$. The right panel shows this measure. Since its mass is concentrated in small vicinities of $\theta=0$ and $\theta=\pi/2$, simultaneous large values of $S_a(T)$ and D are unlikely. This means that (1) the conditional random variable $D|S_a(T)$ and the random variable D have similar distributions so that the fragility of the Bouc-Wen system under consideration is nearly independent of $S_a(T)$ and (2) the response spectrum $S_a(T)$ is an unsatisfactory IM when dealing with large seismic events since large values of D and $S_a(T)$ are nearly independent.

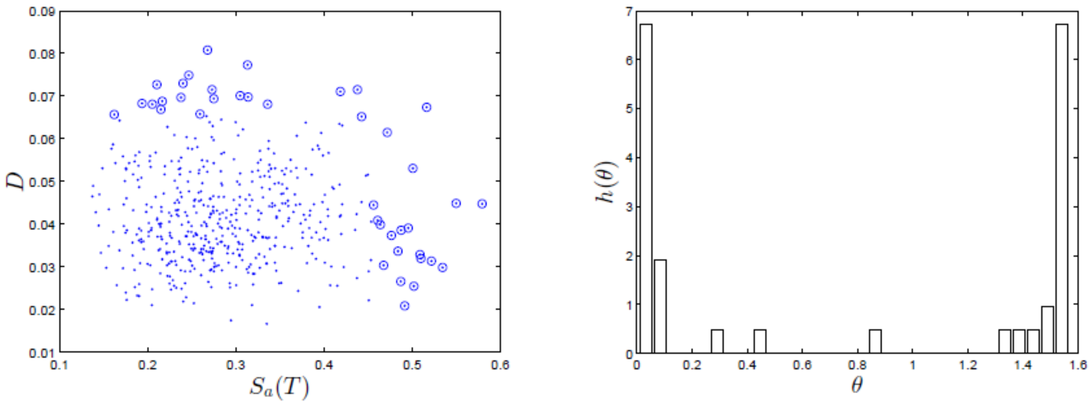

Figure 1. Samples of $(S_a(T),D)$ and estimates of the angular measure $h(\theta)$ (left and right panels)

5 CONCLUSIONS

Fragilities are usually defined as probabilities that structural demand parameters D exceed critical values for given values of intensity measures (IMs), e.g., response spectra $S_a(T)$. Plots of these conditional probabilities versus IMs are essential tools of performance-based earthquake engineering. Intuition suggests that D and $S_a(T)$ are weakly dependent since

these random variables are maxima of responses of linear oscillators and complex nonlinear structural systems to the seismic ground acceleration process $A(t)$.
Concepts of the multivariate extreme value theory (MEVT) have been used to estimate the dependence between D and $S_a(T)$ under the assumption that the probability law of $A(t)$ is known. It was found that the intuition regarding the dependence of D and $S_a(T)$ is correct, e.g., D and $S_a(T)$ are nearly independent for the Bouc-Wen oscillator subjected to large seismic events. This suggests that $S_a(T)$ is an unsatisfactory intensity measure so that the usefulness of fragilities defined as functions of $S_a(T)$ is questionable.

REFERENCES

[1] H. Ebrahimian, F. Jalayer, A. Lucchini, F. Mollaioli, and G. Manfredi. Preliminary ranking of alternative scalar and vector intensity measures of ground shaking. *Bulletin of Earthquake Engineering*, 2015. DOI 10.1007/s10518-015-9755-9.

[2] R.V. Field Jr and M. Grigoriu. A method for the efficient construction and sampling of vector-valued translation random fields. *Probabilistic Engineering Mechanics*, 29:79–91, 2012.

[3] M. Gioffré, V. Gusella, and M. Grigoriu. Simulation of non-Gaussian field applied to wind pressure fluctuations. *Probabilistic Engineering Mechanics*, 5(4):339–346, 2000.

[4] M. Grigoriu. *Applied Non-Gaussian Processes: Examples, Theory, Simulation, Linear Random Vibration, and MATLAB Solutions*. Prentice Hall, Englewoods Cliffs, NJ, 1995.

[5] M. Grigoriu. Do seismic intensity measures (ims) measure up? *Probabilistic Engineering Mechanics*, 46:80–93, 2016.

[6] K. Gurley and A. Kareem. Applications of wavelet transformations in earthquake, wind and ocean engineering. *Engineering Structures*, 21:149–167, 1999.

[7] N.S. Kwong, A.K. Chopra, and R.K. McGuire. Evaluation of ground motion selection and modification procedures using synthetic ground motions. *Earthquake Engineering and Structural Dynamics*, 44:1841–1861, 2015. DOI: 10.1002/eqe.2558.

[8] N.S. Kwong, A.K. Chopra, and R.K. McGuire. A framework for the evaluation of ground motion selection and modification procedures. *Earthquake Engineering and Structural Dynamics*, 44:795–815, 2015. DOI: 10.1002/eqe.2502.

[9] N.S. Kwong, A.K. Chopra, and R.K. McGuire. A ground motion selection procedure for enforcing hazard consistency and estimating seismic demand hazard curves. *Earthquake Engineering and Structural Dynamics*, 44:2467–2487, 2015. DOI: 10.1002/eqe.2593.

[10] M.R. Leadbetter, G.Lindgren, and H.Rootzén. *Extremes and Related Properties of Random Sequences and Processes*. Springer-Verlag, New York, 1983.

[11] A.S. Papageorgiou and K. Aki. A specific barrier model for the quantitative description of the inhomogeneous faulting and the prediction of strong ground motion. Part I: Description of the model. *Bulletin of the Seismilogical Society of America*, pages 693–722, 1983.

[12] S.Resnick. On the foundations of multivariate heavy-tail analysis. *Journal of Applied Probability*, 41:191–212, 2004.

[13] S.Resnick. *Heavy-Tail Phenomena: Probabilistic and Statistical Modeling*. Springer, New York, 2007.